Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE 2009

MARKING SCHEME

APPLIED MATHEMATICS

HIGHER LEVEL

General Guidelines

1. Penalties of three types are applied to a candidate's work as follows:

Slip	– numerical slip	S (–1)
Blunder	– mathematical error	B (–3)
Misreading	– if not serious	M (–1)

2. An 'attempt mark' is awarded as follows: 5 (att 2)

For a serious blunder or omission or misreading which oversimplifies:
– the attempt mark is only awarded.

3. The marking scheme shows one correct solution to each question.
In many cases, there are other equally valid methods.

1. **(a)** A particle is projected vertically upwards from the point p. At the same instant a second particle is let fall vertically from q. The particles meet at r after 2 seconds. The particles have equal speeds when they meet at r.

Prove that $|pr| = 3|rq|$.

$$v = u + ft$$

qr $\quad v = 0 + 2g$

pr $\quad v = u - 2g$

$$\Rightarrow \quad 2v = u$$ [5]

$$v^2 = u^2 + 2fs$$

qr $\quad v^2 = 0 + 2g|qr|$ [5]

pr $\quad v^2 = u^2 - 2g|pr|$

$$v^2 = 4v^2 - 2g|pr|$$

$$3v^2 = 2g|pr|$$ [5]

$$3(2g|qr|) = 2g|pr|$$ [5]

$$3|qr| = |pr|$$ [20]

or

$$v = u + ft$$

qr $\quad v = 0 + 2g$

pr $\quad v = u - 2g$

$$\Rightarrow \quad u = 4g$$ [5]

$$v^2 = u^2 + 2fs$$

qr $\quad 4g^2 = 0 + 2g|qr|$

$$\Rightarrow |qr| = 2g$$ [5]

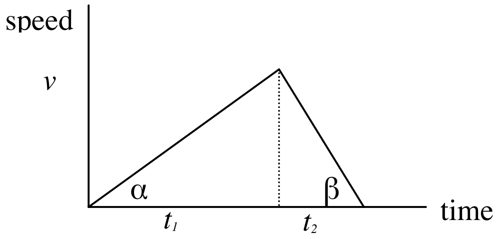
pr $\quad 4g^2 = 16g^2 - 2g|pr|$

$$\Rightarrow |pr| = 6g$$ [5]

$$3|qr| = |pr|$$ [5] [20]

1. **(b)** A train accelerates uniformly from rest to a speed v m/s with uniform acceleration f m/s^2. It then decelerates uniformly to rest with uniform retardation $2f$ m/s^2. The total distance travelled is d metres.

(i) Draw a speed-time graph for the motion of the train.



5,5

(ii) If the average speed of the train for the whole journey is $\sqrt{\frac{d}{3}}$, find the value of f.

$$f = \tan\alpha = \frac{v}{t_1} \Rightarrow t_1 = \frac{v}{f}$$ 5

$$2f = \tan\beta = \frac{v}{t_2} \Rightarrow t_2 = \frac{v}{2f}$$ 5

total time $\quad t_1 + t_2 = \frac{v}{f} + \frac{v}{2f} = \frac{3v}{2f}$

total distance $\quad d = \tfrac{1}{2}(t_1 + t_2)v$ or $t_1 + t_2 = \frac{2d}{v}$

$$t_1 + t_2 = \frac{2d}{v} \Rightarrow \frac{3v}{2f} = \frac{2d}{v} \Rightarrow 3v^2 = 4fd$$ 5

average speed $= \dfrac{\text{total distance}}{\text{total time}}$

$$\sqrt{\frac{d}{3}} = \frac{d}{t_1 + t_2}$$

$$\sqrt{\frac{d}{3}} = \frac{d}{\frac{3v}{2f}} = \frac{2fd}{3v} \quad \text{or} \quad \sqrt{\frac{d}{3}} = \frac{v}{2}$$

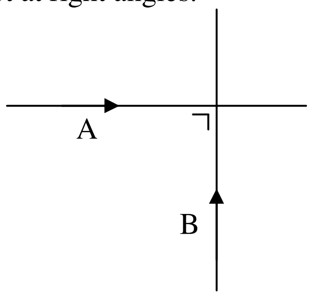
$$\frac{d}{3} = \frac{4f^2d^2}{9v^2} \quad \text{or} \quad \frac{d}{3} = \frac{v^2}{4}$$

$$3v^2 = 4f^2d \quad \text{or} \quad 3v^2 = 4d$$

$$4fd = 4f^2d \quad \text{or} \quad 4fd = 4d$$

$$\Rightarrow f = 1$$ 5 | 30

2. **(a)** Two cars, A and B, travel along two straight roads which intersect at right angles.
A is travelling east at 15 m/s.
B is travelling north at 20 m/s.
At a certain instant both cars are 800 m from the intersection and approaching the intersection.
Find **(i)** the shortest distance between the cars
(ii) the distance each car is from the intersection when they are nearest to each other.



(i)

$$\vec{V}_A = 15\vec{i} + 0\vec{j}$$

$$\vec{V}_B = 0\vec{i} + 20\vec{j}$$

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

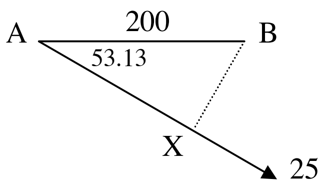
$$= 15\vec{i} - 20\vec{j} \qquad 5$$

$$\text{slope} = -\tfrac{4}{3} \quad \text{or} \quad \text{magnitude} = 25 \text{ m/s} \qquad 5$$

$$\text{or direction: East } 53.13° \text{ South}$$

B reaches intersection in $\frac{800}{20} = 40$ s 5

In this time A travels $15 \times 40 = 600$ m and is now 200 m from the intersection



5

shortest distance $= 200\sin\alpha$

$= 200\sin(53.13°) = 160$ m 5

(ii)

$$\text{time} = 40 + \frac{|AX|}{|\vec{V}_{AB}|}$$

$$= 40 + \frac{200\cos 53.13°}{25}$$

$$= 44.8 \text{ s}$$

In this time A travels $15 \times 44.8 = 672$ m

and B travels $20 \times 44.8 = 896$ m

distance of A from the intersection $= 800 - 672 = 128$ m

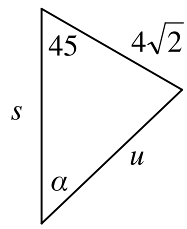
distance of B from the intersection $= 800 - 896 = -96$ m

$= 96$ m past intersection 5 **30**

2 **(b)** The speed of an aeroplane in still air is u km/h.
The aeroplane flies a straight-line course from P to Q, where Q is north of P.

If there is no wind blowing the time for the journey from P to Q is T hours.

Find, in terms of u and T, the time to fly from P to Q if there is a wind blowing from the south-east with a speed of $4\sqrt{2}$ km/h.



$$s = uT$$ 5

$$\frac{\sin\alpha}{4\sqrt{2}} = \frac{\sin 45}{u}$$ 5

$$\sin\alpha = \frac{4}{u} \quad \text{or} \quad u\sin\alpha = 4$$

$$\Rightarrow \cos\alpha = \frac{\sqrt{u^2-16}}{u}$$

$$\text{time} = \frac{s}{u\cos\alpha + 4\sqrt{2}\cos 45}$$ 5

$$= \frac{uT}{\sqrt{u^2-16}+4}$$ 5 **20**

3. **(a)** A straight vertical cliff is 200 m high.
A particle is projected from the top of the cliff.
The speed of projection is $14\sqrt{10}$ m/s at an angle α to the horizontal.
The particle strikes the level ground at a distance of 200 m from the foot of the cliff.

(i) Find, in terms of α, the time taken for the particle to hit the ground.
(ii) Show that the two possible directions of projection are at right angl es to each other.

(i)

$$14\sqrt{10}\cos\alpha . t = 200$$

$$t = \frac{200}{14\sqrt{10}\cos\alpha} \qquad 5$$

(ii)

$$14\sqrt{10}\sin\alpha . t - \tfrac{1}{2}gt^2 = -200 \qquad 5$$

$$14\sqrt{10}\sin\alpha . \left(\frac{200}{14\sqrt{10}\cos\alpha}\right) - \tfrac{1}{2}g\left(\frac{200}{14\sqrt{10}\cos\alpha}\right)^2 = -200 \qquad 5$$

$$200\tan\alpha - \frac{100}{\cos^2\alpha} = -200$$

$$200\tan\alpha - 100\left(1+\tan^2\alpha\right) = -200$$

$$\tan^2\alpha - 2\tan\alpha - 1 = 0$$

$$\tan\alpha = 1 \pm \sqrt{2}$$

$$\tan\alpha_1 \times \tan\alpha_2 = \left(1+\sqrt{2}\right)\left(1-\sqrt{2}\right)$$

$$= -1$$

directions : are perpendicular 5 | 20

3 **(b)** A plane is inclined at an angle $60°$ to the horizontal. A particle is projected up the plane with initial speed u at an angle θ to the inclined plane.
The plane of projection is vertical and contains the line of greatest slope.

The particle strikes the plane at right angles.

Show that the range on the inclined plane is $\dfrac{4\sqrt{3}\,u^2}{13g}$.

$r_j = 0$ **5**

$$0 = u\sin\theta.t - \tfrac{1}{2}g\cos 60.t^2$$

$$\Rightarrow \quad t = \frac{2u\sin\theta}{g\cos 60} \quad \text{or} \quad \frac{4u\sin\theta}{g}$$ **5**

$v_i = 0$

$$0 = u\cos\theta - g\sin 60.t$$

$$\Rightarrow \quad t = \frac{u\cos\theta}{g\sin 60} \quad \text{or} \quad \frac{2u\cos\theta}{g\sqrt{3}}$$ **5**

$$t = \frac{4u\sin\theta}{g} = \frac{2u\cos\theta}{g\sqrt{3}}$$

$$\Rightarrow \tan\theta = \frac{1}{2\sqrt{3}}, \quad \sin\theta = \frac{1}{\sqrt{13}} \quad \text{and} \quad \cos\theta = \frac{2\sqrt{3}}{\sqrt{13}}$$ **5**

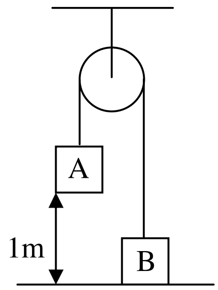
$$\text{Range} = u\cos\theta\left\{\frac{4u\sin\theta}{g}\right\} - \frac{1}{2}g\sin 60.\left\{\frac{4u\sin\theta}{g}\right\}^2$$ **5**

$$= \frac{8u^2\sqrt{3}}{13g} - \frac{4u^2\sqrt{3}}{13g}$$

$$= \frac{4u^2\sqrt{3}}{13g}$$ **5** **30**

4. **(a)** A light inextensible string passes over a small fixed smooth pulley.
A particle A of mass 10 kg is attached to one end of the string and a particle B of mass 5 kg is attached to the other end.

The system is released from rest when B touches the ground and A is 1 m above the ground.



Find **(i)** the speed of A as it hits the ground

(ii) the height that B rises above the horizontal ground.

(i)

$$10g - T = 10f \quad 5$$

$$T - 5g = 5f \quad 5$$

$$5g = 15f$$

$$f = \frac{g}{3} \quad 5$$

A

$$v^2 = u^2 + 2as$$

$$= 0 + 2\left(\frac{g}{3}\right)(1)$$

$$v = \sqrt{\frac{2g}{3}} \text{ or } 2.556 \text{ m/s} \quad 5$$

(ii) B

$$v^2 = u^2 + 2as$$

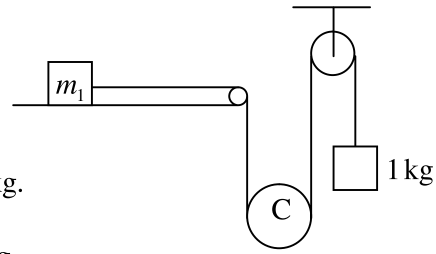
$$0 = \frac{2g}{3} - 2gs$$

$$s = \frac{1}{3}$$

$$\text{height } = 1 + \frac{1}{3} = \frac{4}{3} \text{ m.} \quad 5$$

25

4 **(b)** A mass m_1 kg is at rest on a smooth horizontal table. It is attached to a light inextensible string. The string, after passing over a small fixed pulley at the edge of the table, passes under a small moveable pulley C, of mass m_2 kg. The string then passes over a smooth fixed pulley and supports a mass of 1 kg.



The system is released from rest.

(i) Find, in terms of m_1 and m_2, the tension in the string.

(ii) The pulley C will remain at rest if $\dfrac{2}{m_2} - \dfrac{1}{m_1} = k$.

Find the value of k.

(*i*) m_1	$T = m_1(p)$	5
1 kg	$T - g = 1(q)$	5
m_2	$m_2 g - 2T = m_2\left\{\dfrac{p+q}{2}\right\}$	5
	$m_2 g - 2T = \dfrac{m_2}{2}\left\{\dfrac{T}{m_1} + T - g\right\}$	
	$\dfrac{3m_2 g}{2} = 2T + \dfrac{m_2 T}{2m_1} + \dfrac{m_2 T}{2}$	
	$3m_1 m_2 g = 4m_1 T + m_2 T + m_1 m_2 T$	
	$T = \dfrac{3m_1 m_2 g}{4m_1 + m_2 + m_1 m_2}$	5
(ii) C will remain at rest if $m_2 g - 2T = 0$ or if $\dfrac{p+q}{2} = 0$		
	$m_2 g = \dfrac{6m_1 m_2 g}{4m_1 + m_2 + m_1 m_2}$	
	$4m_1 + m_2 + m_1 m_2 = 6m_1$	
	$m_1 m_2 = 2m_1 - m_2$	
	$1 = \dfrac{2}{m_2} - \dfrac{1}{m_1}$	
	$\Rightarrow k = 1$	5

25

5. **(a)** A smooth sphere P, of mass m kg, moving with speed $2u$ m/s collides directly with a smooth sphere Q, of mass $2m$ kg, moving in the same direction with speed u m/s.
The coefficient of restitution between the spheres is e.

(i) Find, in terms of e, the speed of each sphere after the collision.

(ii) Prove that the speed of Q increases after the collision.

(iii) Find the value of e if the speed of P after the collision is $\frac{10u}{9}$ m/s.

(*i*) PCM $m(2u) + 2m(u) = mv_1 + 2mv_2$ — 5

NEL $v_1 - v_2 = -e(2u - u)$ — 5

$v_1 = \frac{u(4-2e)}{3}$ — 5

$v_2 = \frac{u(4+e)}{3}$ — 5

(*ii*) $v_2 = \frac{u(4+e)}{3}$

$> u$ as $e > 0$

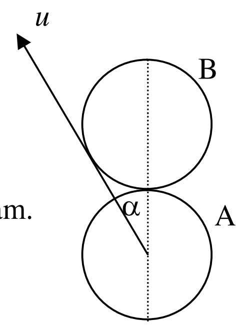
$\Rightarrow$ speed of Q increases — 5

(*iii*) $v_1 = \frac{u(4-2e)}{3}$

$\frac{10u}{9} = \frac{u(4-2e)}{3}$

$\Rightarrow e = \frac{1}{3}$ — 5 — 30

5 **(b)** A smooth sphere A, of mass m kg, moving with speed u, collides with a stationary identical smooth sphere B.
The direction of motion of A, before impact, makes an angle α with the line of centres at impact and just touches sphere B, as shown in the diagram.
The coefficient of restitution between the spheres is $\frac{4}{5}$.



(i) Show that $\alpha = 30°$.

(ii) Find the direction in which each sphere travels after the collision.

(iii) Find the percentage loss in kinetic energy due to the col lision.

(i) $\sin\alpha = \dfrac{r}{2r} \Rightarrow \alpha = 30°$ — 5

(ii) PCM $\quad m\left(\dfrac{u\sqrt{3}}{2}\right) + m(0) = mv_1 + mv_2$

NEL $\quad v_1 - v_2 = -\dfrac{4}{5}\left(\dfrac{u\sqrt{3}}{2} - 0\right)$ — 5

$$\Rightarrow v_1 = \frac{u\sqrt{3}}{20} \text{ and } v_2 = \frac{9u\sqrt{3}}{20}$$

$$\text{velocity of A} = -\frac{u}{2}\vec{i} + \frac{u\sqrt{3}}{20}\vec{j}$$

$$\text{direction of A} = \tan^{-1}\left(\frac{\sqrt{3}}{10}\right)$$

$$\text{velocity of B} = 0\vec{i} + \frac{9u\sqrt{3}}{20}\vec{j}$$

direction of B = along line of centres — 5

(iii)

$$\text{KE before} = \tfrac{1}{2}mu^2$$

$$\text{KE after} = \tfrac{1}{2}m\left\{\frac{u^2}{4} + \frac{3u^2}{400} + \frac{243u^2}{400}\right\}$$

$$\text{KE lost} = \tfrac{27}{400}mu^2$$

$$\text{\% KE lost} = \frac{\frac{27}{400}mu^2}{\frac{1}{2}mu^2} \times 100 = 13.5\%$$ — 5 — 20

6. **(a)** The distance, x, of a particle from a fixed point, o, is given by

$$x = a\cos(\omega t + \varepsilon)$$

where a, ω and ε are constants.

(i) Show that the motion of the particle is simple harmonic.

A particle moving with simple harmonic motion starts from a point 5 cm from the centre of the motion with a speed of 1 cm/s.

(ii) The period of the motion is 11 seconds. Find the maximum speed of the particle, correct to two decimal places.

(i)

$$x = a\cos(\omega t + \varepsilon)$$

$$\dot{x} = -a\omega\sin(\omega t + \varepsilon) \qquad 5$$

$$\ddot{x} = -a\omega^2\cos(\omega t + \varepsilon)$$

$$= -\omega^2 x$$

$$\Rightarrow \text{S.H.M. about } x = 0. \qquad 5$$

(ii) Period = 11

$$\frac{2\pi}{\omega} = 11$$

$$\omega = \frac{2\pi}{11} \text{ or } \frac{4}{7} \qquad 5$$

$$x = 5,\ t = 0 \ \Rightarrow 5 = a\cos\varepsilon$$

$$v = 1,\ t = 0 \ \Rightarrow 1 = -a\omega\sin\varepsilon$$

$$\cos\varepsilon = \frac{5}{a} \Rightarrow \sin\varepsilon = \frac{\sqrt{a^2 - 25}}{a}$$

$$\Rightarrow 1 = -a\left(\frac{4}{7}\right)\frac{\sqrt{a^2 - 25}}{a}$$

$$a = 5.3$$

or

$$v = \omega\sqrt{a^2 - x^2}$$

$$1 = \frac{4}{7}\sqrt{a^2 - 25}$$

$$a = 5.3 \qquad 5$$

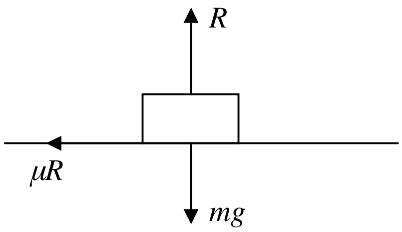
$$v_{\max} = \omega a = \frac{4}{7} \times 5.3$$

$$= 3.03 \text{ cm/s.} \qquad 5$$

25

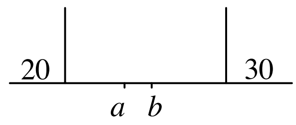
6 **(b)** A table moves in a horizontal plane with simple harmonic motion. The table completes N oscillations per minute.

Find, in terms of μ and N, the greatest allowable amplitude of the motion if an object placed on the table is not to slip, where μ is the coefficient of friction.



$\text{frequency} = \dfrac{N}{60}$	5	
$\dfrac{\omega}{2\pi} = \dfrac{N}{60}$		
$\omega = \dfrac{\pi N}{30}$	5	
$F = mr\omega^2$		
$\mu R = mr\omega^2$	5	
$\mu mg = mr\omega^2$	5	
$\mu g = r\left(\dfrac{\pi N}{30}\right)^2$		
$r = \dfrac{900\mu g}{\pi^2 N^2}$ or $\dfrac{893.65\mu}{N^2}$	5	25

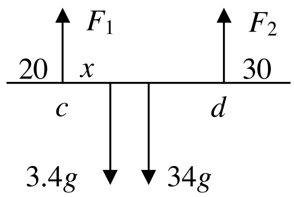
7. **(a)** A uniform rod of length 2 m and of mass 34 kg, is suspended by two vertical strings.



One string is attached to a point 20 cm from one end and can just support a mass of 17 kg without breaking; the second string is attached 30 cm from the other end and can just support a mass of 20.74 kg without breaking.

A mass of 3.4 kg is now attached to the rod.

Find the length of the section *ab* of the rod within which the 3.4 kg mass can be attached without breaking either string.



Take moments about c :

$$F_2(1.5) = 3.4g(x) + 34g(0.8)$$ 5

$$20.74g(1.5) = 3.4g(x) + 34g(0.8)$$

$$\Rightarrow x = 1.15 \text{ m}$$ 5

Take moments about d :

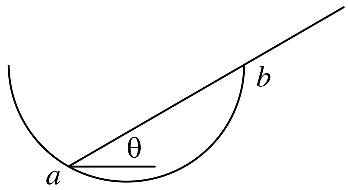
$$F_1(1.5) = 3.4g(1.5 - x) + 34g(0.7)$$ 5

$$17g(1.5) = 3.4g(1.5 - x) + 34g(0.7)$$

$$\Rightarrow x = 1 \text{ m}$$ 5

$$\Rightarrow |ab| = 0.15 \text{ m or } 15 \text{ cm}$$ 5 | 25

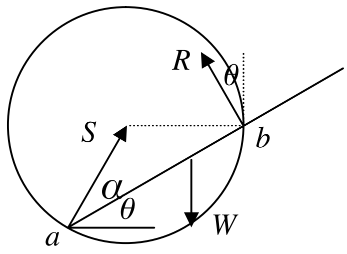
7 **(b)** A uniform rod of length $2p$ and weight W rests with its lower end a in contact with a smooth hemispherical bowl, of radius p. The axis of the bowl is vertical.



The upper end of the rod projects beyond the rim of the bowl as shown in the diagram. The inclination of the rod to the horizontal is θ . The point b on the rod is in contact with the rim of the bowl. $|ab| = 2p\cos\theta$.

(i) Find, in terms of W, the reaction at b.

(ii) Show that $\cos\theta = 2\cos 2\theta$.



(i) Moments about a :

$$R(2p\cos\theta) = W(p\cos\theta)$$ 5

$$R = \frac{W}{2}$$ 5

(ii) $$\alpha = \theta$$ 5

horiz : $R\sin\theta = S\cos 2\theta$

$$\Rightarrow S = \frac{R\sin\theta}{\cos 2\theta} = \frac{W\sin\theta}{2\cos 2\theta}$$ 5

vert : $R\cos\theta + S\sin 2\theta = W$

$$\left(\frac{W}{2}\right)\cos\theta + \left(\frac{W\sin\theta}{2\cos 2\theta}\right)\sin 2\theta = W$$

$$\cos 2\theta\cos\theta + \sin 2\theta\sin\theta = 2\cos 2\theta$$

$$\cos(2\theta - \theta) = 2\cos 2\theta$$

$$\cos\theta = 2\cos 2\theta$$ 5 25

8. **(a)** Prove that the moment of inertia of a uniform rod of mass m and length 2ℓ about an axis through its centre perpendicular to the rod is $\frac{1}{3}m\ell^2$.

Let $\text{M} = $ mass per unit length

mass of element $= \text{M}\{\text{dx}\}$

moment of inertia of the element $= \text{M}\{\text{dx}\}\text{x}^2$ 5

moment of inertia of the rod $= \text{M} \int_{\ell}^{\ell} \text{x}^2\, \text{dx}$ 5

$= \text{M}\left[\dfrac{\text{x}^3}{3}\right]_{-\ell}^{\ell}$ 5

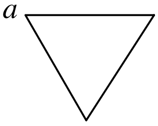
$= \frac{2}{3}\text{M}\,\ell^3$

$= \frac{1}{3}\text{m}\,\ell^2$ 5 20

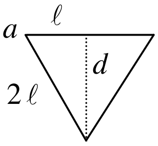
8 **(b)** Three equal uniform rods, each of length 2ℓ and mass m, form the sides of an equilateral triangle *abc*.

(i) Find the moment of inertia of the frame *abc* about an axis through *a* perpendicular to the plane of the triangle.

The triangular frame *abc* is attached to a smooth hinge at *a* about which it can rotate in a vertical plane. The frame is held with *ab* horizontal, and *c* below *ab*, and then released from rest.



(ii) Find the maximum angular velocity of the triangle in the subsequent motion.



(i)

$$(2\ell)^2 = \ell^2 + d^2$$

$$d = \ell\sqrt{3}$$ 5

$$\begin{aligned} I &= \tfrac{4}{3}m\ell^2 + \tfrac{4}{3}m\ell^2 + \left\{\tfrac{1}{3}m\ell^2 + md^2\right\} \\ &= 3m\ell^2 + md^2 \\ &= 3m\ell^2 + m\left(\ell\sqrt{3}\right)^2 \\ &= 6m\ell^2 \end{aligned}$$ 5

(ii) Gain in KE = Loss in PE

$$\tfrac{1}{2}I\omega^2 = Mgh$$

$$= 3mgh$$ 5

$$h = \tfrac{1}{3}d$$ 5

$$\tfrac{1}{2}\left(6m\ell^2\right)\omega^2 = (3m)g\left(\tfrac{1}{3}d\right)$$ 5

$$3\ell^2\omega^2 = g\left(\ell\sqrt{3}\right)$$

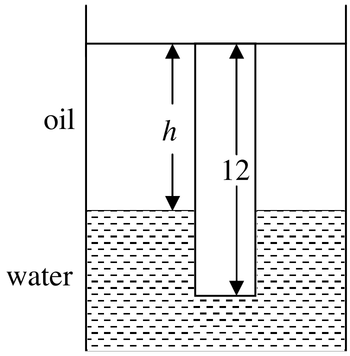
$$\omega = \sqrt{\frac{g\sqrt{3}}{3\ell}}$$ 5

30

9. **(a)** A uniform cylindrical piece of wood 12 cm long floats in water with its axis vertical and 10 cm of its length immersed.

Oil of relative density 0.75 is poured on to the water until the top of the cylinder is in the surface of the oil.

Find the depth of the layer of oil.



water $B = W$

$$\frac{\frac{10}{12}W(1)}{s} = W$$

relative density of rod $s = \frac{5}{6}$ 5

Let the depth of oil $= h$

$$B_{oil} + B_{water} = W$$ 5

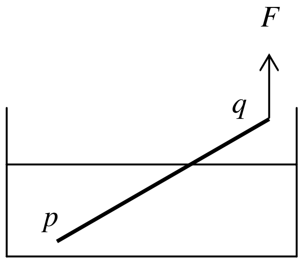
$$\frac{\left(\frac{h}{12}\right)W(0.75)}{\frac{5}{6}} + \frac{\left(\frac{12-h}{12}\right)W(1)}{\frac{5}{6}} = W$$ 5,5

$$\left(\frac{h}{12}\right)(0.75) + \left(\frac{12-h}{12}\right) = \frac{5}{6}$$

$$0.75h + 12 - h = 10$$

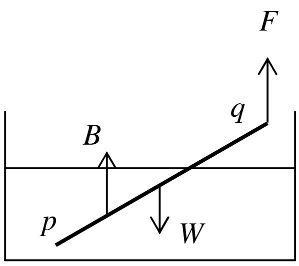
$\Rightarrow h = 8$ cm. 5 | 25

9 **(b)** A thin uniform rod *pq* of weight *W* is in equilibrium in an inclined position with end *p* immersed in a container of water. The end *q* is supported by a vertical force *F*, as shown in the diagram. The relative density of the material of the rod is *s*.



(i) Find in terms of *s* the fraction of the length of the rod that is immersed.

(ii) If $s = \frac{3}{4}$, find *F* in terms of *W*.



(*i*)	Let length of immersed part $= x$		
	Take moments about q :		
	$B\left(\ell - \frac{x}{2}\right)\sin\theta = W\,\tfrac{1}{2}\ell\sin\theta$	5,5	
	$B = \frac{\frac{x}{\ell}W(1)}{s} = \frac{xW}{\ell s}$	5	
	$\left(\frac{xW}{\ell s}\right)\left(\ell - \frac{x}{2}\right)\sin\theta = W\,\tfrac{1}{2}\ell\sin\theta$		
	$x^2 - 2\ell x + \ell^2 s = 0$		
	$x = \frac{2l \pm \sqrt{4\ell^2 - 4\ell^2 s}}{2}$		
	fraction $= \frac{x}{\ell} = 1 - \sqrt{1-s}$	5	
(*ii*)	$\frac{x}{\ell} = 1 - \sqrt{1 - \tfrac{3}{4}} = \frac{1}{2}$		
	$\Rightarrow B = \frac{W}{2s} = \frac{2W}{3}$		
	$B + F = W$		
	$\frac{2W}{3} + F = W$		
	$\Rightarrow F = \frac{W}{3}$	5	25

10. **(a)** Solve the differential equation

$$\frac{dy}{dx} = \frac{1}{xy} + \frac{y}{x}$$

given that $y = \sqrt{3}$ when $x = 1$.

$$\frac{dy}{dx} = \frac{1}{xy} + \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{1+y^2}{xy}$$ 5

$$\int \frac{y}{1+y^2}\,dy = \int \frac{1}{x}\,dx$$ 5

$$\frac{1}{2}\ln\left(1+y^2\right) = \ln x + C$$ 5

$$y = \sqrt{3},\ x = 1$$

$$\Rightarrow\ C = \frac{1}{2}\ln 4 \text{ or } \ln 2$$ 5

$$\frac{1}{2}\ln\left(1+y^2\right) = \ln x + \ln 2$$

$$\ln\left(1+y^2\right)^{\frac{1}{2}} = \ln 2x$$

$$\Rightarrow\ 1 + y^2 = 4x^2$$

$$\Rightarrow\ y = \sqrt{4x^2 - 1}$$ 5 | 25

10 **(b)** A particle of mass m is projected vertically upwards with speed u. The air resistance is kv^2 per unit mass when the speed is v.

The maximum height reached by the particle is $\frac{\ln 4}{2k}$.

(i) Find the value of u in terms of k.

(ii) Find the value of k if the time to reach the greatest height is $\frac{\pi}{3}$ seconds.

(*i*)

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

$$-mg - mkv^2 = m\frac{vdv}{dx} \qquad 5$$

$$v\frac{dv}{dx} = -\left(g + kv^2\right)$$

$$-\int_u^0 \frac{v}{g + kv^2}\, dv = \int_0^{\frac{\ln 4}{2k}} dx \qquad 5$$

$$\left[-\frac{1}{2k}\ln\left(g + kv^2\right)\right]_u^0 = \frac{\ln 4}{2k} \qquad 5$$

$$-\frac{1}{2k}\ln(g) + \frac{1}{2k}\ln\left(g + ku^2\right) = \frac{\ln 4}{2k}$$

$$\ln\left(\frac{g + ku^2}{g}\right) = \ln 4$$

$$\Rightarrow u = \sqrt{\frac{3g}{k}} \qquad 5$$

(*ii*)

$$\frac{dv}{dt} = -\left(g + kv^2\right)$$

$$-\int_u^0 \frac{1}{g + kv^2}\, dv = \int_0^{\frac{\pi}{3}} dt$$

$$\left[-\frac{1}{k}\frac{1}{\sqrt{\frac{g}{k}}}\tan^{-1}\left(\frac{v}{\sqrt{\frac{g}{k}}}\right)\right]_{\sqrt{\frac{3g}{k}}}^0 = \frac{\pi}{3}$$

$$\frac{1}{\sqrt{gk}}\tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$$

$$\Rightarrow k = \frac{1}{g} \qquad 5 \qquad 25$$

Coimisiún na Scrúduithe Stáit

Marcanna Breise as ucht freagairt trí Ghaeilge

Léiríonn an tábla thíos an méid marcanna breise ba chóir a bhronnadh ar iarrthóirí a ghnóthaíonn níos mó ná 75% d'iomlán na marcanna.

N.B. Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don scrúdú. Ba chóir freisin an marc bónais sin **a shlánú síos**.

Tábla 300 @ 5%

Bain úsáid as an tábla seo i gcás na n-ábhar a bhfuil 300 marc san iomlán ag gabháil leo agus inarb é 5% gnáthráta an bhónais.

Bain úsáid as an ngnáthráta i gcás 225 marc agus faoina bhun sin. Os cionn an mharc sin, féach an tábla thíos.

Bunmharc	Marc Bonais
226	11
227 - 233	10
234 - 240	9
241 - 246	8
247 - 253	7
254 - 260	6

Bunmharc	Marc Bonais
261 - 266	5
267 - 273	4
274 - 280	3
281 - 286	2
287 - 293	1
294 - 300	0